Applied Soft Computing 142 (2023) 110336

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Computational-based partitioning and Strong (α, β)-cut based novel method for intuitionistic fuzzy time series forecasting

Manish Pant [a,*], Kamlesh Bisht [b], Seema Negi [c]

[a] Department of Mathematics, Invertis University, Bareilly, Uttar Pradesh, 263145, India
[b] Department of Mathematics, Uttarakhand Open University Haldwani, Nainital 263139, Uttarakhand, India
[c] Department of Mathematics, S.V. Govt. P.G. College, Lohaghat 262524, Uttarakhand, India

ARTICLE INFO

Article history:
Received 30 November 2022
Received in revised form 16 March 2023
Accepted 13 April 2023
Available online 24 April 2023

Keywords:
Intuitionistic fuzzy time series
Computational-based partitioning
Strong (α, β)-cut
Intuitionistic fuzzy logical relationship
Forecasting

ABSTRACT

In this article, we propose a computational-based partitioning (CBP) and Strong (α, β)-cut based novel intuitionistic fuzzy time series (IFTS) forecasting method. Construction of intervals, intuitionistic fuzzification of time series data, appropriate intuitionistic fuzzy logical relationships (IFLRs), and procedure of defuzzification are critical issues that affect the forecasting accuracy of any IFTS method. A Computational-based partitioning approach uses basic statistical parameters to determine the number of intervals and constructing of intervals without specialized knowledge of the domain. For intuitionistic fuzzification of time series data, all those intuitionistic fuzzy sets are taken having both non-zero membership and non-membership grade. A Strong (α, β)-cut are used to choose apposite IFLRs that deliver importance in investigating the tendency of time series data. We also propose a defuzzification approach to get the numerical values. In this article, two popular historical time series datasets are used to demonstrate the supremacy of the proposed forecasting method. Root mean square error (RMSE) and mean absolute percentage error (MAPE) are used to verify the performance of the proposed method. Verification of the validity of the proposed forecasting method is authenticated by using the values of the Theil inequality coefficient (U) and tracking signal (TS).

1. Introduction

Time series forecasting is the process of analyzing historical time-stamped data using statistics and modeling to make predictions. Applications of time series forecasting are widely used in healthcare, metrology, economics, engineering, sociology, finance, and in several fields of management and science. For any type of data processing, uncertainty is an essential aspect which is also a significant component of time series data. Regression analysis, autoregressive integrated moving average, exponential smoothing and weighted moving average based on various conventional time series forecasting model have been developed to model the probabilistic uncertainty efficiently but they fail to address the issue of hidden uncertainties that arises because of non-probabilistic reasons.

Zadeh [1] introduced the fuzzy set theory to handle uncertainty in the system caused by non-probabilistic reasons of imprecision, incompleteness, ambiguity, and linguistic representation of time series data. Song and Chissom [2–4] developed fuzzy set-based time-invariant and time-variant fuzzy time series (FTS) forecasting models to handle the limitations of conventional models for time series forecasting. Chen [5] developed fuzzy logical relations (FLR) and simple arithmetic operations based robust FTS forecasting model to forecast enrollment at the University of Alabama (EUA). Huarng [6] presented a model using average and distribution-based heuristic approaches and also address the importance of the construction of intervals in terms of interval length of any FTS forecasting model to affect the forecasting accuracy. Teoh et al. [7] and Cheng et al. [8] developed a cumulative probability distribution approach (CPDA) and fuzzy clustering-based financial FTS forecasting methods to determine interval length. Huarng and Yu [9], Chen and Chung [10], and Kuo et al. [11] proposed the integration of soft computing that incorporates neural networks, genetic algorithms, and particle swarm optimization techniques in FTS forecasting to obtain the length of intervals. Afterward, many researchers [12–16] developed FTS forecasting methods based on these soft computing techniques.

Several methods have been looked into for FLR identification, which is a crucial step in correctly modeling FTS. Many researchers [2–4] have used a single matrix to establish FLR. Chen [5] opted to employ a simpler technique that involves grouping FLR. After then, researchers frequently used the aforementioned technique. Sullivan and Woodall [17] and Huarng

* Corresponding author.
E-mail addresses: mani.pant728@gmail.com (M. Pant), kamlesh45848@gmail.com (K. Bisht), seema.negi999@gmail.com (S. Negi).

https://doi.org/10.1016/j.asoc.2023.110336
1568-4946/

and Yu [18] proposed a model, transition matrix, and the feed-forward artificial neural network has been used to identification of fuzzy relation in FTS forecasting. Recently, Goyal and Bisht [19] proposed a Strong alpha cut-based FTS forecasting model for the identification of fuzzy relations.

Atanassov [20] explained the limitation of a fuzzy set as non-determinacy occurs in the system using a single function for both degrees of membership and non-membership. They generalized the fuzzy set to an intuitionistic fuzzy set (IFS) using two separate functions to determine the degree of membership and non-membership to model non-determinacy in the system. Joshi and Kumar [21] were the first to develop IFS based FTS forecasting method to deal with non-determinacy in time series data of EUA. Kumar and Gangwar [22] defined intuitionistic fuzzy time series (IFTS) and developed the IFTS forecasting method to forecast the market share price of the State Bank of India (SBI) and the time series data of EUA.

In general, the complete procedure of the IFTS forecasting method in the following sequence: Defining the Universe of discourse (UOD); partitioning of UOD into intervals or construction of intervals; defining the fuzzy sets and IFSs and intuitionistic fuzzification of experimental data; determining the IFLRs and their groups on fuzzified experimental data; defuzzification of forecasted fuzzy output to obtain the numeric output. Construction of intervals, intuitionistic fuzzification of time series data, apposite IFLRs, and procedure of intuitionistic defuzzification are important factors for any IFTS model that impact the forecasting accuracy of the model. Joshi and Kumar [21] and Kumar and Gangwar [22] used predefining the construction of equal-length intervals. Gangwar and Kumar [23] also developed a forecasting model based on IFS in which the partition of UOD into unequal-length intervals is done using CPDA. Wang et al. [24] developed the IFTS forecasting method and used the fuzzy clustering technique to partition UOD into unequal-length intervals. Abhishek et al. [25,26] and Gautam and Singh [27] proposed an IFTS forecasting model for intuitionistic fuzzified time series data using the score and accuracy function of an intuitionistic fuzzy number. Recently, Pant et al. [28] and Pant and Kumar [29] used an interpolating polynomial as an objective function and PSO to optimize the length of intervals for the IFTS forecasting method.

The general process to establish IFLRs in the IFTS forecasting method define $I_t \rightarrow I_{t+1}$ where I_t and I_{t+1} are the intuitionistic fuzzified output of the current state (t) and next state (t+1), respectively. In the aforementioned IFTS models, IFLRs are established only using those IFSs having a membership grade maximum. Egrioglu et al. [30], Kocak et al. [31,32], and Pattanayak et al. [33] use pi-sigma artificial neural networks, robust regression, long short-term memory, and support vector machine to define IFLRs in IFTS forecasting. The selection of intuitionistic defuzzification also affects IFTS forecasting. Many researchers [27,34–36] commonly used the weighted average formula for intuitionistic defuzzification. Abhishek et al. [25] proposed a unique defuzzification technique in which the defuzzify value lies in intervals. Bisht and Kumar [37] proposed IFS based computational method and concluded that there is no need for defuzzification in this method. Sanjay Kumar [38] introduced a modified weighted average formula for defuzzification to get crisp output. Recently, Pant and Kumar [39] gave an optimized weighted average defuzzification approach via the grey wolf optimization algorithm.

The aforesaid IFTS forecasting models make the simple calculations but they use a number of the partitioning of UOD taking it arbitrarily and including only those IFSs having membership grade is maximum and does not consider non-membership grade without any reason in the procedure of intuitionistic fuzzification and establish IFLRs. In defuzzification approach considers those IFSs having the highest membership grade. In this case, it still requires expert knowledge and presumption to partition UOD and less information is passed to intuitionistic fuzzification of time series data and to establish IFLRs and to defuzzification which might mislead the forecasted results. Therefore, motivated by these limitations, an IFTS forecasting method based on novel approaches for partitioning of UOD, intuitionistic fuzzification, establishing apposite IFLRs, and defuzzification is proposed. The main contributions of this research article are the following:

(1) Computational-based partitioning (CBP) approach uses basic statistical parameters to determine the number of intervals and constructing of intervals. This CPB approach does not need an expert of domain knowledge of the data and a novice user is also capable of creating intervals.
(2) For intuitionistic fuzzification of time series data considered all those IFSs have both non-zero membership and non-membership grades.
(3) Apposite IFLRs are constructed into account all inputs that may have an impact on forecasting accuracy. Strong (α, β)-cut based approaches are used to choose these IFLRs.
(4) Lastly, intuitionistic defuzzify forecast output to get crisp values based on both non-zero membership and non-membership values.

To demonstrate the superiority of the proposed CPB and Strong (α, β)-cut based IFTS forecasting method to forecast two popular historical time series datasets are used which are EUA and the market share price of State Bank of India (SBI) at the Bombay Stock Exchange (BSE), India. Performance analysis and validity of proposed method using error measures (RMSE and MAPE) and validation test (U and TS). Diebold Mariano test is used to test the significant difference between the performance of two different forecasting methods.

The rest of the contents of this research article is organized as follows: In Section 2, we present basic preliminaries related to the proposed method. The proposed methodology is presented in Section 3. Application and comparative experimental analysis of the proposed forecasting method are explained in Sections 4 and 5. The results of the Diebold Mariano test are explained in Sect.6. Finally, Section 7 concludes the finding of the present work.

2. Preliminaries

This section provides definitions of fuzzy set, fuzzy time series (FTS), Intuitionistic fuzzy set (IFS), and Intuitionistic fuzzy time series (IFTS), and are defined as follows.

Definition 1. Let $X = \left\{x_j \,|1 \le j \le n\right\}$ be a finite discrete set. A fuzzy set F on X is mathematically defined as below:

$$F = \left\{\left(x_j,\ \mu_F(x_j)\right) \left|x_j \in X\right.\right\}$$

Here, $\mu_F : X \rightarrow [0, 1]$, and $\mu_F\left(x_j\right)$ represents membership degree of x_j in fuzzy set F.

Definition 2. Let fuzzy sets $f_j(t)$ $\forall j \in \mathbb{N}$ are defined on $X\,(t) \subseteq \mathbb{R}$ $\forall t \in \mathbb{Z}$. A Collection of fuzzy sets $f_j(t)$ is known as FTS $F(t)$ defined on $X\,(t)$.

Definition 3. Let $X = \left\{x_j \,|1 \le j \le n\right\}$ be a finite discrete set. An IFS I on X is mathematically defined as below:

$$I = \left\{\left(x_j,\ \mu_I(x_j),\ \upsilon_I(x_j)\right) : x_j \in X\right\}$$

Here, μ_I and υ_I are membership and non-membership functions of an IFS I such that $\mu_I, \upsilon_I : X \rightarrow [0, 1]$ with condition $0 \le \mu_I(x_j) + \upsilon_I(x_j) \le 1$ $\forall x_j \in X$. Where $\mu_I\left(x_j\right)$ and $\upsilon_I\left(x_j\right)$ represents

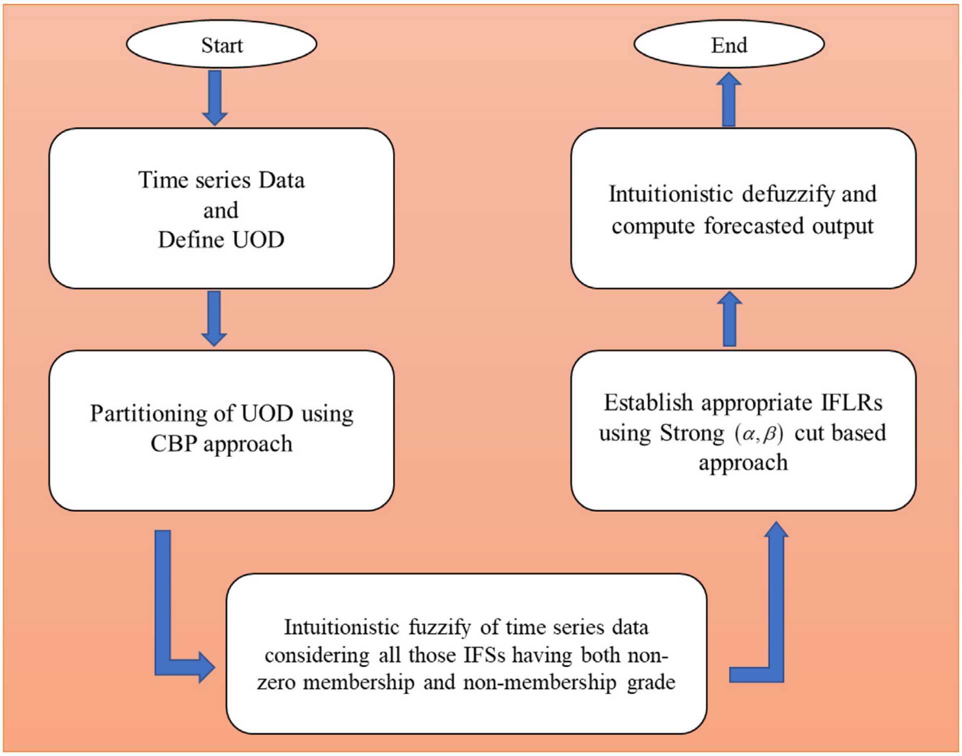

Fig. 1. Flowchart of proposed CBP and Strong (α, β)-cut based IFTS forecasting method.

the grade of membership and non-membership x_j in I. The value of $\pi_I = 1 - \mu_I(x_j) - \upsilon_I(x_j)$ is called the degree of indeterminacy or non-determinacy of the $x_j \in X$ to the IFS I. If $\pi_I\left(x_j\right) = 0, \forall x_j \in X$ then IFS I, reduces to a fuzzy set.

Definition 4. Let $X(t) \subseteq \mathbb{R}\ \forall t \in \mathbb{Z}$ be the universe of discourse. A Collection $\xi(t)$ of IFSs $I_j(t)\ \forall j \in \mathbb{N}$ is called IFTS $\xi(t)$on $X(t)$.

3. Proposed CBP and strong (α, β)-cut based IFTS forecasting method

In this segment, we present a stepwise description of the proposed CBP and Strong (α, β)-cut based intuitionistic fuzzy time series forecasting method. The flowchart of the proposed IFTS forecasting is shown in Fig. 1.

Step 1: Specify the Universe of discourse (UOD) as $X = \left[X_{low} - X_1, X_{hig} + X_2\right]$ where X_{low} and X_{hig} are lowest and highest values of given time series data. Here, X_1 and X_2 two positive real numbers must be taken in order to fit whole time series data.

Step 2: Computational-based partitioning (CBP) approach used to determine the number of intervals and partition of the UOD. Following steps of the CBP approach are as follows.

Step 2.1: Calculating mean (λ) and standard deviation (σ) of given time series data and $p_0 = \lambda$ is used as the center of intervals.

Step 2.2: To find out the number of points to the left of p_0 is denoted as the left end (m_l) and is calculated as

$$m_l = \frac{2 * (p_0 - X_{low})}{\sigma} + \frac{(X_{hig} + X_{low})}{(X_{hig} - X_{low})} \tag{1}$$

Similarly, find out the number of points to the right of p_0 is denoted as the right end (m_r) and is calculated as

$$m_r = \frac{2 * (X_{hig} - p_0)}{\sigma} + \frac{(X_{hig} + X_{low})}{(X_{hig} - X_{low})} \tag{2}$$

Step 2.3: Using the following two equations calculate the points to the left and right of p_0 respectively.

$$p_{-i} = p_0 - \frac{i\sigma}{2} i = 1, 2, 3, \ldots, fl(m_l) \tag{3}$$

$$p_i = p_0 + \frac{i\sigma}{2} i = 1, 2, 3, \ldots, fl(m_r) \tag{4}$$

Where $fl(m_l)$ and $fl(m_r)$ is the floor function of m_l and m_r.

Step 2.4: Compute the number of intervals (n) using the following equation.

$$n = fl\,(m_l) + fl\,(m_r) \tag{5}$$

Step 2.5: Partition of UOD into n equal-length intervals and create n intervals as

$$u_j = \left[p_{j-(i+1)}, p_{j-i}\right], j = 1, 2, \ldots, n, i = \frac{n}{2} \tag{6}$$

Step 3: Define triangular fuzzy sets in accordance with intervals and fuzzify time series data.

Step 4: Following construction method of Singh et al. [40] is used to construct IFSs from a fuzzy set.

Let A_F be any fuzzy set on X. Then $A_I = \left\{\left\langle x_j, f(\mu_{A_I}(x_j))\right\rangle : \forall x_j \in X\right\}$ is an IFS. *i.e* $A_I = \left\{\left\langle x_j, f_\mu(\mu_{A_I}(x_j)), f_\upsilon(\upsilon_{A_I}(x_j))\right\rangle : \forall x_j \in X\right\}$ with $f_\mu\left(\mu_{A_I}(x_j)\right) = \mu_{A_I}(x_j)$ and $f_\upsilon\left(\upsilon_{A_I}(x_j)\right) = g\left(1 - \mu_{A_I}(x_j)\right)$. Here $g : [0, 1] \to [0, 1]$ is non-membership function or indeterminacy function and is defined as $g(x_j) = \frac{\mu_{A_I}(x_j)}{\sum_{j=1}^{j=n} \mu_{A_I}(x_j)} * \left(1 - \mu_{A_I}(x_j)\right)$ with condition $g(0) = 0$ and $g(x_j) \leq \mu_{A_I}(x_j)$. where n is the total number of data points.

According to the construct IFSs in this step, each datum of time series will lie in either one or more membership functions and non-membership functions along with non-zero membership and non-membership values. Intuitionistic fuzzify time series data having non-zero membership and non-membership grades.

Step 5: Use the following steps of Strong (α, β)-cut approach to establish IFLRs between two successive data points at a time $(t-1)$ and t.

(1) Obtain a set S of all possible relations from X to Y, where

$$\left.\begin{aligned} &S = \left\{ \left(X^R Y, \mu_{X \cdot Y}, \upsilon_{X \cdot Y}\right) \middle| X^R Y \right. \\ &\qquad \left. \text{are all possible relations from } X \text{ to } Y \right\} \\ &\mu_{X \cdot Y}\left(x_j, x_k\right) = \max\left(\mu_X(x_j), \mu_Y(x_k)\right) \text{ and} \\ &\upsilon_{X \cdot Y}\left(x_j, x_k\right) = \min\left(\upsilon_X(x_j), \upsilon_Y(x_k)\right) \end{aligned}\right\} \tag{7}$$

Where, x_j and x_k are two successive data points.

If $X^{R_i} Y$ is ith relation between X and Y and denoted by $\left(X^{R_i} Y, {\mu^i}_{X \cdot Y}, {\upsilon^i}_{X \cdot Y}\right)$ where ${\mu^i}_{X \cdot Y}\left(x_j, x_k\right) = \max\left({\mu^i}_X(x_j), {\mu^i}_Y(x_k)\right)$ and ${\upsilon^i}_{X \cdot Y}\left(x_j, x_k\right) = \min\left({\upsilon^i}_X(x_j), {\upsilon^i}_Y(x_k)\right)$

(2) Compute the values of α and β as

$$\left.\begin{aligned} &\alpha_t = \frac{\sum_{i=1}^{|S|}\left(\max\left({\mu^i}_X(x_j), {\mu^i}_Y(x_k)\right)\right)}{|S|} \text{ and} \\ &\alpha = \min\left(\alpha_t : 2 \le t \le n\right) \\ &\beta_t = \frac{\sum_{i=1}^{|S|}\left(\min\left({\upsilon^i}_X(x_j), {\upsilon^i}_Y(x_k)\right)\right)}{|S|} \text{ and} \\ &\beta = \max\left(\beta_t : 2 \le t \le n\right) \end{aligned}\right\} \tag{8}$$

Where, $|S|$ and n are the cardinality of set S and the end number of data respectively.

(3) Establish IFLRs using the following Strong (α, β)-cut set T.

$$\text{Strong } (\alpha, \beta) \text{ -cut set } T = \left\{\left(X^R Y, \mu_{X \cdot Y}, \upsilon_{X \cdot Y}\right) \middle| \left(X^R Y, \mu_{X \cdot Y}, \upsilon_{X \cdot Y}\right) \in S \text{ such that } \mu_{X \cdot Y} > \alpha, \upsilon_{X \cdot Y} < \beta\right\}$$

So, choose IFLRs are such that $X \to Y \text{s.t.} X^R Y \in T$

For example, if for time $(t-1)$ and t intuitionistic fuzzified datum are $\{I_1, 0.52, 0.40\}, \{I_2, 0.48, 0.50\}$ and $\{I_3, 0.62, 0.26\}, \{I_4, 0.38, 0.48\}$ then

(1) The Set of all possible relations is

$$S = \left\{\left({I_1}^R I_3, 0.62, 0.26\right), \left({I_1}^R I_4, 0.52, 0.40\right), \left({I_2}^R I_3, 0.62, 0.26\right), \left({I_2}^R I_4, 0.48, 0.48\right)\right\}$$

(2) Once all sets S for time series data are calculated then calculating $\alpha = 0.50, \beta = 0.45$

(3) Strong (α, β)set $T = \left\{\left({I_1}^R I_3, 0.62, 0.26\right), \left({I_1}^R I_4, 0.52, 0.40\right), \left({I_2}^R I_3, 0.62, 0.26\right)\right\}$

So, choose IFLRs are $I_1 \to I_3, I_1 \to I_4$ and $I_2 \to I_3$

Step 6: Intuitionistic defuzzify forecast output to get a numerical value for time t using the following rule with non-zero membership and non-membership values:

Rule 1: If there are only one IFLRs for time t which is shown as $I_i \to I_j$ then forecasted output for time t is m_j. Where, m_j is the upper value of the interval u_j.

Rule 2: If there are more than one IFLRs for time t then the forecasted output for time t is calculated as

$$FD_t = \frac{\sum_{i=1}^{m}\left(\mu_i - \upsilon_i\right) m_j}{\sum_{i=1}^{m}\left(\mu_i - \upsilon_i\right)}, j = 1, 2, \ldots, m \tag{9}$$

Where m is the cardinality of Strong (α, β)set T, μ_i, υ_i and m_j are the membership and non-membership grade of the antecedent I_i and upper value of interval u_j of resultant I_j for IFLR$I_i \to I_j$, respectively.

4. Error analysis and validation of the proposed method

Using the root mean square error (RMSE), mean absolute percentage error (MAPE), correlation coefficient (R), coefficient of determination (R^2), Theil inequality coefficient (U), and tracking signal (TS) the proposed forecasting method's effectiveness and validity are checked. Expressions of RMSE, MAPE, R, R^2, U, and TS are shown as follows.

1 RMSE	$\sqrt{\frac{1}{n_f}\sum_{i=1}^{n_f}(AD_i - FD_i)^2}$
2. MAPE	$\frac{1}{n_f}\sum_{i=1}^{n_f}\frac{\lvert FD_i - AD_i\rvert}{AD_i} \times 100\%$
3. R	$\frac{n_f \sum AD_i FD_i - \left(\sum AD_i\right)\left(\sum FD_i\right)}{\sqrt{n_f\left(\sum {AD_i}^2\right) - \left(\sum AD_i\right)^2}\sqrt{n_f\left(\sum {FD_i}^2\right) - \left(\sum FD_i\right)^2}}$
4. U	$\frac{RMSE}{\sqrt{\frac{1}{n_f}\sum_{i=1}^{n_f} {AD_i}^2} + \sqrt{\frac{1}{n_f}\sum_{i=1}^{n_f} {FD_i}^2}}$
5. TS	$\frac{\sum_{i=1}^{n_f}(FD_i - AD_i)}{\frac{1}{n_f}\sum_{i=1}^{n_f}\lvert FD_i - AD_i\rvert}$

n_f = forecasted data points, AD_i and FD_i are actual and forecasted ith time series datum.

R is a statistical measure of the strength of the relationship between the two variables that range from −1 and 1. R with a negative or positive value implies a negative or positive relationship between two variables. U is very useful for comparing different forecasting methods and it varies between 0 and 1. The values of U are closer to 0 and 1 implying that perfect match and no match between actual and forecasted data. TS determines the presence of biasness of a forecasting model along with the tendency of above or under forecast. The minimum and maximum value of TS at −4 and 4 implies above and under forecast with biasness.

5. Simulation of the proposed method

In this segment, the proposed forecasting method is applied to the historical benchmark enrollment of the University of Alabama (EUA) and the market price of a share of State Bank of India (SBI) at the Bombay Stock Exchange (BSE), India time series datasets. These two-time series datasets are considered for the comparative study of the proposed IFTS forecasting method since they are frequently used to assess the performance accuracy of the fuzzy time series forecasting method.

5.1. Forecasting of enrollment of the University of Alabama

Step 1: By observing the maximum and minimum value of EUA from Table 1 and two positive real numbers X_1 and X_2 are taken as 410.27 and 406.63, UOD is specified as $X = [12644.73, 19743.63]$.

Step 2: UOD is partitioned into intervals using the following steps.

Step 2.1: Mean (λ) and standard deviation (σ) of the time series of EUA are 16194.18 and 1774.72 and $p_0 = \lambda$ is taken as the center of intervals.

Step 2.2: Left end (m_l) and right end (m_r) are calculated as

$$m_l = \frac{2 * (1619.18 - 13055)}{1774.72} + \frac{(19337 + 13055)}{(19337 - 13055)} = 8.6939$$

And $m_r = \frac{2 * (19337 - 1619.18)}{1774.72} + \frac{(19337 + 13055)}{(19337 - 13055)} = 8.6980$

Table 1
Actual enrollments of the University of Alabama from year 1971 to 1992.

Year	Enrollment	Year	Enrollment
1971	13055	1982	15433
1972	13563	1983	15497
1973	13868	1984	15145
1974	14696	1985	15163
1975	15460	1986	15984
1976	15311	1987	16859
1977	15603	1988	18150
1978	15861	1989	18970
1979	16807	1990	19328
1980	16919	1991	19337
1981	16388	1992	18876

Step 2.3: Points to the left and right of p_0 are calculated using Eqs.3 and Eqs.4:

$p_{-8} = 12644.73, p_{-7} = 13088.41,$

$p_{-6} = 13532.09, p_{-5} = 13975.77$

$p_{-4} = 14419.45, p_{-3} = 14863.14, p_{-2} = 15306.82,$

$p_{-1} = 15750.50$

And $p_1 = 16637.86, p_2 = 17081.54, p_3 = 17525.22, p_4 = 17968.91$
$p_5 = 18412.59, p_6 = 18856.27, p_7 = 19299.95, p_8 = 19743.63$

Step 2.4: The number of intervals (n) are calculated as 16 using Eqs.5.

Step 2.5: UOD is partitioned into 16 intervals and creates 16 intervals as

$u_1 = [12644.73, 13088.41], u_2 = [13088.41, 13532.09],$

$u_3 = [13532.09, 13975.77], u_4 = [13975.77, 14419.45]$

$u_5 = [14419.45, 14863.14], u_6 = [14863.14, 15306.82],$

$u_7 = [15306.82, 15750.50], u_8 = [15750.50, 16194.18]$

$u_9 = [16194.18, 16637.86], u_{10} = [16637.86, 17081.54],$

$u_{11} = [17081.54, 17525.22], u_{12} = [17525.22, 17968.91]$

$u_{13} = [17968.91, 18412.59], u_{14} = [18412.59, 18856.27],$

$u_{15} = [18856.27, 19299.95], u_{16} = [19299.95, 19743.63]$

Step 3: Following sixteen fuzzy sets are defined in accordance of intervals.

$$F_i = \begin{cases} [12644.73 + (i-1)h,\ 12644.73 + i{*}h,\ 12644.73 + 2{*}i{*}h], \\ \ i = 1 \text{ to } 15,\ h = 443.68 \\ [12644.73 + (i-1)h,\ 12644.73 + i{*}h,\ 12644.73 + i{*}h], \\ \ i = 16, h = 443.68 \end{cases}$$

The membership grades to fuzzy sets are shown in Table 2.

Step 4: The construction algorithm of Singh et al. [40] is implemented to construct sixteen IFSs (I_i) from fuzzy sets F_i and each datum is intuitionistic fuzzified into related IFSs having non-zero membership and non-membership grades. The intuitionistic fuzzified EUA is shown in Table 3.

Step 5: A set of all possible relations and values of α_t and β_t for each enrollment datum are calculated using Eqs.7 and Eqs.8, respectively which is shown in Table 4. Then values of α and β are calculated and Strong (α, β)-cut set T are constructed and IFLRs are established between two succeeding data points and shown in Table 5.

Step 6: Intuitionistic defuzzification procedures (Rules 1 and 2) are implemented to forecast the EUA. An illustration to forecast the enrollment for the year 1973 is explained as follows.

For enrollment of the year 1973, datum 13867 belongs to IFSs in I_2 and I_3. 0.2451 and 0.7548 are the membership grade for enrollment 13867 in I_2 and I_3 while 0.1574 and 0.2244 are the non-membership grade for enrollment 13867 in I_2 and I_3. Hence intuitionistic fuzzified enrollment for the year 1973 represented as $\{I_2, 0.2451, 0.1574\}$ and $\{I_3, 0.7548, 0.2244\}$. For forecasting the enrollment for the year 1973, intuitionistic fuzzified enrollment for the year 1972 (13563) is used i.e., $\{I_2, 0.9303, 0.0551\}$ and $\{I_3, 0.0696, 0.0786\}$.

(1) The Set of all possible relations is

$$S = \begin{Bmatrix} \left(I_2{}^R I_2, 0.9303, 0.0551\right), \left(I_2{}^R I_3, 0.9303, 0.0551\right), \\ \left(I_3{}^R I_2, 0.2451, 0.0786\right), \left(I_3{}^R I_3, 0.7548, 0.0786\right) \end{Bmatrix}$$

Table 2
Membership grade for enrollments to fuzzy sets.

Enrollments	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}	F_{16}
13055	0.9246	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13563	0	0.9303	0.0696	0	0	0	0	0	0	0	0	0	0	0	0	0
13868	0	0.2451	0.7548	0	0	0	0	0	0	0	0	0	0	0	0	0
14696	0	0	0	0.3767	0.6232	0	0	0	0	0	0	0	0	0	0	0
15460	0	0	0	0	0	0.6547	0.3452	0	0	0	0	0	0	0	0	0
15311	0	0	0	0	0	0.9905	0.0094	0	0	0	0	0	0	0	0	0
15603	0	0	0	0	0	0.3324	0.6675	0	0	0	0	0	0	0	0	0
15861	0	0	0	0	0	0	0.7509	0.249	0	0	0	0	0	0	0	0
16807	0	0	0	0	0	0	0	0	0.6187	0.3812	0	0	0	0	0	0
16919	0	0	0	0	0	0	0	0	0.3663	0.6336	0	0	0	0	0	0
16388	0	0	0	0	0	0	0	0.5631	0.4368	0	0	0	0	0	0	0
15433	0	0	0	0	0	0.7156	0.2843	0	0	0	0	0	0	0	0	0
15497	0	0	0	0	0	0.5713	0.4286	0	0	0	0	0	0	0	0	0
15145	0	0	0	0	0.3647	0.6352	0	0	0	0	0	0	0	0	0	0
15163	0	0	0	0	0.3241	0.6758	0	0	0	0	0	0	0	0	0	0
15984	0	0	0	0	0	0	0.4737	0.5262	0	0	0	0	0	0	0	0
16859	0	0	0	0	0	0	0	0	0.5015	0.4984	0	0	0	0	0	0
18150	0	0	0	0	0	0	0	0	0	0	0	0.5918	0.4081	0	0	0
18970	0	0	0	0	0	0	0	0	0	0	0	0	0	0.7436	0.2563	0
19328	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.9367	0.0632
19337	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.9164	0.0835
18876	0	0	0	0	0	0	0	0	0	0	0	0	0	0.9555	0.0444	0

Table 3
Intuitionistic fuzzified enrollments dataset.

Enrollments	Intuitionistic fuzzified with associate membership and non-membership grade
13055	$\{I_1, 0.9246, 0.0753\}$
13563	$\{I_2, 0.9303, 0.0551\}$, $\{I_3, 0.0696, 0.0786\}$
13868	$\{I_2, 0.2451, 0.1574\}$, $\{I_3, 0.7548, 0.2244\}$
14696	$\{I_4, 0.3767, 0.6232\}$, $\{I_5, 0.6232, 0.1789\}$
15460	$\{I_6, 0.6547, 0.0494\}$, $\{I_7, 0.3452, 0.0763\}$
15311	$\{I_6, 0.9905, 0.0020\}$, $\{I_7, 0.0094, 0.0031\}$
15603	$\{I_6, 0.3324, 0.0484\}$, $\{I_7, 0.6675, 0.0749\}$
15861	$\{I_7, 0.7509, 0.0631\}$, $\{I_8, 0.2491, 0.1397\}$
16807	$\{I_9, 0.6187, 0.1226\}$, $\{I_{10}, 0.3812, 0.1558\}$
16919	$\{I_9, 0.3663, 0.1206\}$, $\{I_{10}, 0.6336, 0.1533\}$
16388	$\{I_8, 0.5631, 0.1837\}$, $\{I_9, 0.4368, 0.1278\}$
15433	$\{I_6, 0.7156, 0.0444\}$, $\{I_7, 0.2843, 0.0687\}$
15497	$\{I_6, 0.5713, 0.0535\}$, $\{I_7, 0.4286, 0.0827\}$
15145	$\{I_5, 0.3647, 0.1765\}$, $\{I_6, 0.6352, 0.0506\}$
15163	$\{I_6, 0.3241, 0.1669\}$, $\{I_7, 0.6758, 0.0478\}$
15984	$\{I_7, 0.4737, 0.0842\}$, $\{I_8, 0.5262, 0.1862\}$
16859	$\{I_9, 0.5015, 0.1299\}$, $\{I_{10}, 0.4984, 0.1652\}$
18150	$\{I_{12}, 0.5918, 0.4081\}$, $\{I_{13}, 0.4081, 0.5914\}$
18970	$\{I_{14}, 0.7436, 0.1121\}$, $\{I_{15}, 0.2563, 0.0884\}$
19328	$\{I_{15}, 0.9367, 0.0274\}$, $\{I_{16}, 0.0632, 0.4036\}$
19337	$\{I_{15}, 0.9164, 0.0355\}$, $\{I_{16}, 0.0835, 0.5216\}$
18876	$\{I_{14}, 0.9555, 0.0250\}$, $\{I_{15}, 0.0445, 0.0197\}$

Table 4
Set of all possible relations and values α_t and for β_t for enrollments data.

Enrollments	Set of all possible relations	α_t	β_t
13055	–	–	–
13563	$\{I_1^R I_2, 0.9303, 0.0551\}$, $\{I_1^R I_3, 0.9246, 0.0753\}$	0.9275	0.0652
13868	$\{I_2^R I_2, 0.9303, 0.0551\}$, $\{I_2^R I_3, 0.9303, 0.0551\}$, $\{I_3^R I_2, 0.2451, 0.0786\}$, $\{I_3^R I_3, 0.7548, 0.0786\}$	0.7151	0.0668
14696	$\{I_2^R I_4, 0.3767, 0.1574\}$, $\{I_2^R I_5, 0.6232, 0.1574\}$, $\{I_3^R I_4, 0.7548, 0.2244\}$, $\{I_3^R I_5, 0.7548, 0.1789\}$	0.6274	**0.1795**
15460	$\{I_4^R I_6, 0.6547, 0.0494\}$, $\{I_4^R I_7, 0.3767, 0.0763\}$, $\{I_5^R I_6, 0.6547, 0.0494\}$, $\{I_5^R I_7, 0.6232, 0.0763\}$	0.5773	0.0628
15311	$\{I_6^R I_6, 0.9905, 0.0020\}$, $\{I_6^R I_7, 0.6547, 0.0020\}$, $\{I_7^R I_6, 0.9905, 0.0031\}$, $\{I_7^R I_7, 0.3452, 0.0031\}$	0.7452	0.0025
15603	$\{I_6^R I_6, 0.9905, 0.0020\}$, $\{I_6^R I_7, 0.9905, 0.0020\}$, $\{I_7^R I_6, 0.3324, 0.0031\}$, $\{I_7^R I_7, 0.6675, 0.0031\}$	0.7452	0.0025
15861	$\{I_6^R I_7, 0.7509, 0.0484\}$, $\{I_6^R I_8, 0.7509, 0.0484\}$, $\{I_7^R I_7, 0.3324, 0.0631\}$, $\{I_7^R I_8, 0.6675, 0.0749\}$	0.6254	0.0587
16807	$\{I_7^R I_9, 0.7509, 0.0631\}$, $\{I_7^R I_{10}, 0.7509, 0.0631\}$, $\{I_8^R I_9, 0.6187, 0.1226\}$, $\{I_8^R I_{10}, 0.3812, 0.1397\}$	0.6254	0.0971
16919	$\{I_9^R I_9, 0.6187, 0.1206\}$, $\{I_9^R I_{10}, 0.6336, 0.1206\}$, $\{I_{10}^R I_9, 0.3812, 0.1226\}$, $\{I_{10}^R I_{10}, 0.6336, 0.1533\}$	0.5668	0.1293
16388	$\{I_9^R I_8, 0.5621, 0.1206\}$, $\{I_9^R I_9, 0.4368, 0.1206\}$, $\{I_{10}^R I_8, 0.6336, 0.1536\}$, $\{I_{10}^R I_9, 0.6336, 0.1278\}$	0.5668	0.1306
15433	$\{I_8^R I_6, 0.7156, 0.0444\}$, $\{I_8^R I_7, 0.5631, 0.0687\}$, $\{I_9^R I_6, 0.7156, 0.0444\}$, $\{I_9^R I_7, 0.4368, 0.0687\}$	0.6078	0.0566
15497	$\{I_6^R I_6, 0.7156, 0.0444\}$, $\{I_6^R I_7, 0.7156, 0.0444\}$, $\{I_7^R I_6, 0.5713, 0.0535\}$, $\{I_7^R I_7, 0.4286, 0.0687\}$	0.6078	0.0528
15145	$\{I_6^R I_5, 0.5713, 0.0535\}$, $\{I_6^R I_6, 0.6352, 0.0535\}$, $\{I_7^R I_5, 0.4286, 0.0827\}$, $\{I_7^R I_6, 0.6352, 0.0506\}$	0.5676	0.0601
15163	$\{I_5^R I_5, 0.3647, 0.1669\}$, $\{I_5^R I_6, 0.6758, 0.0478\}$, $\{I_6^R I_5, 0.6358, 0.0506\}$, $\{I_6^R I_6, 0.6352, 0.0478\}$	0.5879	0.0783
15984	$\{I_5^R I_7, 0.4337, 0.0872\}$, $\{I_5^R I_8, 0.5262, 0.1669\}$, $\{I_6^R I_7, 0.6758, 0.0478\}$, $\{I_6^R I_8, 0.6758, 0.0478\}$	0.5879	0.0867
16859	$\{I_7^R I_9, 0.5015, 0.0872\}$, $\{I_7^R I_{10}, 0.4984, 0.0872\}$, $\{I_8^R I_9, 0.5262, 0.1299\}$, $\{I_8^R I_{10}, 0.5262, 0.1652\}$	**0.5131**	0.1159
18150	$\{I_9^R I_{12}, 0.5918, 0.1299\}$, $\{I_9^R I_{13}, 0.5015, 0.1299\}$, $\{I_{10}^R I_{12}, 0.5918, 0.1652\}$, $\{I_{10}^R I_{13}, 0.4984, 0.1652\}$	0.5459	0.1475
18970	$\{I_{12}^R I_{14}, 0.7436, 0.1121\}$, $\{I_{12}^R I_{15}, 0.5918, 0.0884\}$, $\{I_{13}^R I_{14}, 0.7436, 0.1121\}$, $\{I_{13}^R I_{15}, 0.4081, 0.0884\}$	0.6218	0.1003
19328	$\{I_{14}^R I_{15}, 0.9667, 0.0274\}$, $\{I_{14}^R I_{16}, 0.7436, 0.1121\}$, $\{I_{15}^R I_{15}, 0.9667, 0.0274\}$, $\{I_{15}^R I_{16}, 0.2563, 0.0884\}$	0.7183	0.0639
19337	$\{I_{15}^R I_{15}, 0.9667, 0.0274\}$, $\{I_{15}^R I_{16}, 0.9667, 0.0274\}$, $\{I_{16}^R I_{15}, 0.9164, 0.0355\}$, $\{I_{16}^R I_{16}, 0.0835, 0.4036\}$	0.7183	0.1235
18876	$\{I_{15}^R I_{14}, 0.9555, 0.0250\}$, $\{I_{15}^R I_{15}, 0.9164, 0.0197\}$, $\{I_{16}^R I_{14}, 0.9555, 0.0250\}$, $\{I_{16}^R I_{15}, 0.0835, 0.0197\}$	0.7277	0.0223

Table 5
IFLRs for enrollments dataset.

Enrollments	IFLRs	Enrollments	IFLRs
13055	–	15433	$I_8 \to I_6, I_8 \to I_7, I_9 \to I_6$
13563	$I_1 \to I_2, I_1 \to I_3$	15497	$I_6 \to I_6, I_6 \to I_7, I_7 \to I_6$
13868	$I_2 \to I_2, I_2 \to I_3, I_3 \to I_3$	15145	$I_6 \to I_5, I_6 \to I_6, I_7 \to I_6$
14696	$I_2 \to I_5, I_3 \to I_5$	15163	$I_6 \to I_6, I_7 \to I_5, I_7 \to I_6$
15460	$I_4 \to I_6, I_5 \to I_6, I_5 \to I_7$	15984	$I_5 \to I_8, I_6 \to I_7, I_6 \to I_8$
15311	$I_6 \to I_6, I_6 \to I_7, I_7 \to I_6$	16859	$I_8 \to I_9, I_8 \to I_{10}$
15603	$I_6 \to I_6, I_6 \to I_7, I_7 \to I_7$	18150	$I_9 \to I_{12}, I_{10} \to I_{12}$
15861	$I_6 \to I_7, I_6 \to I_8, I_7 \to I_8$	18970	$I_{12} \to I_{14}, I_{12} \to I_{15}, I_{13} \to I_{14},$
16807	$I_7 \to I_9, I_7 \to I_{10}, I_8 \to I_9$	19328	$I_{14} \to I_{15}, I_{14} \to I_{16}, I_{15} \to I_{15}$
16919	$I_9 \to I_9, I_9 \to I_{10}, I_{10} \to I_{10}$	19337	$I_{15} \to I_{15}, I_{15} \to I_{16}, I_{16} \to I_{15}$
16388	$I_9 \to I_8, I_{10} \to I_8, I_{10} \to I_9$	18876	$I_{15} \to I_{14}, I_{15} \to I_{15}, I_{16} \to I_{14}$

$$FD_{1973} = \frac{(0.9303 - 0.0551) * 13532.09 + (0.9303 - 0.0551) * 13975.77 + (0.7548 - 0.0786) * 13975.77}{(0.9303 - 0.0551) + (0.9303 - 0.0551) + (0.7548 - 0.0786)}$$
$$= 13875.15$$

Box I.

Table 6
Forecasted enrollments of University of Alabama.

Enrollments	Joshi & Kumar [21]	Kumar & Gangwar [22]	Wang et al. [24]	Abhishek et al. [25]	Bisht & Kumar [37]	Abhishek et al. [26]	Gautam & Singh [27]	Pant & Kumar [29]	Pant & Kumar [39]	Proposed Method
13055	–	–	–	–	–	–	–	–	–	–
13563	14250	13693	13500	13500	–	13500	13423	13682	–	13750.61
13867	14246	13693	14155	14250	–	13800	13668	13682	–	13815.75
14696	14246	14867	14155	14250	14850	14400	14648	14722	14527.334	14863.14
15460	15491	15287	15539	15499.6	15358	15300	15383	15427	15376.77	15444.88
15311	15491	15376	15539	15374.6	15300	15300	15309	15544	15208.76	15416.99
15603	15491	15376	15502	15500	15750	15300	15546	15544	15206.31	15584.46
15861	16345	15376	15502	15374.6	15750	16200	15309	15544	15829.72	16038.15
16807	16345	16523	16667	16666.6	16650	16200	16748	16665	16407.91	16800.89
16919	15850	16606	16667	17000	17225	17000	17178	15994	16463.17	16933.35
16388	15850	17519	15669	17000	16257	17000	17178	17230	16481.48	16351.27
15433	15850	16606	15564	15500	15300	15300	15383	15994	15275.02	15426.25
15497	15450	15376	15564	15374.6	15720	15300	15309	15544	15385.86	15466.9
15145	15450	15376	15564	15374.6	14893	15300	15309	15544	15226	15170.41
15163	15491	15287	15523	15499.6	15300	15450	15546	15516	15385.86	15165.89
15984	15491	15287	15223	15499.6	15634	15450	15546	15516	15936.23	16021.69
16859	16345	16523	16799	16666.6	16590	16800	16748	16665	16799.5	16849.38
18150	17950	17519	18268	17000	18000	17000	17178	17230	17913.24	17968.91
18970	18961	19500	18268	18500	18900	18900	18962	18820	19115.06	18982.71
19328	18961	19000	18780	20000	19350	19200	19208	19311	18959.25	19414.3
19337	18961	19500	19575	19250	19350	18900	19208	19311	19069.65	19449.39
18876	18961	19500	18855	19250	19264	18900	18593	19311	18773.05	19000.54

(2) Values of α_t and β_t calculated as

$$\alpha_t = \frac{(0.9303 + 0.9303 + 0.2451 + 0.7548)}{4} = 0.7151$$

$$\beta_t = \frac{(0.0551 + 0.0551 + 0.0786 + 0.0786)}{4} = 0.0668$$

Similarly, α_t and β_t $(2 \le t \le n)$ are calculated for each datum and $\alpha = \min(\alpha_t : 2 \le t \le n) = 0.51$ and $\beta = \max(\beta_t : 2 \le t \le n) = 0.17$ are calculated from Table 4.

(3) Strong (α, β)set $T = \left\{ \left(I_2{}^R I_2, 0.9303, 0.0551 \right), \left(I_2{}^R I_3, 0.9303, 0.0551 \right), \left(I_3{}^R I_3, 0.7548, 0.0786 \right) \right\}$

So, for enrollment for the year 1973, IFLRs are $I_2 \to I_2, I_2 \to I_3$ and $I_3 \to I_3$.

Intuitionistic defuzzify forecast output to get forecasted value enrollment for the year 1973 using rule 2 because there are more than one IFLRs for enrollment for the year 1973. See Box I.

The rest of the other enrollments are also forecasted in a similar manner and shown in Table 6. Table 6 also shows the forecasted enrollment using various IFS-based forecasting methods suggested by [21,22,24–27,29,37,39]. Table 7 presents a comparison between the performance of the proposed forecasting method with these existing IFS-based forecasting methods in forecasting enrollments in terms of statistical error measure.

Table 7
Comparison between performance of proposed and different methods in forecasting enrollments.

Models	RMSE	MAPE	U	TS	R	R^2
Joshi & Kumar [21]	433.76	2.243	0.0132	−4.85	0.9688	0.9385
Kumar & Gangwar [22]	493.56	2.336	0.015	1.483	0.9594	0.9204
Wang et al. [24]	350.9	1.805	0.0107	−5.49	0.977	0.9545
Abhishek et al. [25]	414.25	1.883	0.0126	−2.819	0.9702	0.9412
Bisht & Kumar [37]	195.57	1.002	0.0058	−0.75	0.9922	0.9844
Abhishek et al. [26]	382.03	1.641	0.0116	−12.47	0.9767	0.9539
Gautam & Singh [27]	347.89	1.456	0.0106	−7.67	0.9796	0.9596
Pant & Kumar [29]	422.68	1.872	0.0128	−0.491	0.9681	0.9372
Pant & Kumar [39]	226.35	1.10	0.0058	−12.85	0.9925	0.985
Proposed Method	**93.14**	**0.41**	**0.0028**	**1.78**	**0.9936**	**0.9872**

Table 8
Actual market share price of SBI from month 2008 to month 2010.

Months	SBI Prices	Months	SBI Prices
8/Apr	1819.95	9/Apr	1355
8/May	1840	9/May	1891
8/Jun	1496.7	9/Jun	1935
8/Jul	1567.5	9/Jul	1840
8/Aug	1638.9	9/Aug	1886.9
8/Sep	1618	9/Sep	2235
8/Oct	1569.9	9/Oct	2500
8/Nov	1375	9/Nov	2394
8/Dec	1325	9/Dec	2374.75
9/Jan	1376.4	10/Jan	2315.25
9/Feb	1205.9	10/Feb	2059.95
9/Mar	1132.25	10/Mar	2120.05

5.2. Forecasting market share price of SBI at BSE

Forecasting stock price volatility entails substantial risk and is essential for investors seeking a profitable investment. Due to its inherent character, financial time series data have long been regarded as the best approach to simulate any forecasting method. In order to evaluate the suitability and applicability of the proposed forecasting method in financial time series forecasting, the market share price of SBI at BSE has been forecasted. Time series data of the market share price of SBI from April 2008 to March 2010 are shown in Table 8.

Maximum and minimum of the market share price of SBI are observed and UOD is defined as $X = [1004.30, 2568.35]$. UOD of

Table 9
Membership grade for market share price of SBI.

Enrollments	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8
1819.95	0	0	0	0.828	0.1719	0	0	0
1840	0	0	0	0.7254	0.2745	0	0	0
1496.7	0	0.4814	0.5185	0	0	0	0	0
1567.5	0	0.1193	0.8806	0	0	0	0	0
1638.9	0	0	0.754	0.2459	0	0	0	0
1618	0	0	0.8609	0.139	0	0	0	0
1569.9	0	0.107	0.8929	0	0	0	0	0
1375	0.1039	0.896	0	0	0	0	0	0
1325	0.3596	0.6403	0	0	0	0	0	0
1376.4	0.09676	0.9032	0	0	0	0	0	0
1205.9	0.9688	0.0311	0	0	0	0	0	0
1132.25	0.6544	0	0	0	0	0	0	0
1355	0.2062	0.7937	0	0	0	0	0	0
1891	0	0	0	0.4646	0.5353	0	0	0
1935	0	0	0	0.2395	0.7604	0	0	0
1840	0	0	0	0.7254	0.2745	0	0	0
1886.9	0	0	0	0.4855	0.5144	0	0	0
2235	0	0	0	0	0	0.705	0.2949	0
2500	0	0	0	0	0	0	0.3496	0.6503
2394	0	0	0	0	0	0	0.8917	0.1082
2374.25	0	0	0	0	0	0	0.9928	0.0071
2315.25	0	0	0	0	0	0.2945	0.7054	0
2059.95	0	0	0	0	0.6004	0.3995	0	0
2120.05	0	0	0	0	0.293	0.7069	0	0

Table 10
Intuitionistic fuzzified market share price of SBI dataset.

SBI prices	Intuitionistic fuzzified with associate membership and non-membership grade	SBI prices	Intuitionistic fuzzified with associate membership and non-membership grade
1819.95	$\{I_4, 0.8281, 0.0369\}, \{I_5, 0.1719, 0.0415\}$	1355	$\{I_1, 0.2062, 0.0684\}, \{I_2, 0.7938, 0.0412\}$
1840	$\{I_4, 0.7254, 0.0516\}, \{I_5, 0.2746, 0.0581\}$	1891	$\{I_4, 0.4646, 0.0645\}, \{I_5, 0.5354, 0.0726\}$
1496.7	$\{I_2, 0.4814, 0.0628\}, \{I_3, 0.5185, 0.0638\}$	1935	$\{I_4, 0.2395, 0.0472\}, \{I_5, 0.7605, 0.0531\}$
1567.5	$\{I_2, 0.1193, 0.0264\}, \{I_3, 0.8806, 0.0268\}$	1840	$\{I_4, 0.7254, 0.0516\}, \{I_5, 0.2746, 0.0581\}$
1638.9	$\{I_3, 0.7540, 0.0474\}, \{I_4, 0.2460, 0.0481\}$	1886.9	$\{I_4, 0.4855, 0.0648\}, \{I_5, 0.5145, 0.0729\}$
1618	$\{I_3, 0.8609, 0.0306\}, \{I_4, 0.1391, 0.0310\}$	2235	$\{I_6, 0.7050, 0.0987\}, \{I_7, 0.2950, 0.0642\}$
1569.9	$\{I_2, 0.1070, 0.0240\}, \{I_3, 0.8930, 0.0244\}$	2500	$\{I_7, 0.3496, 0.0702\}, \{I_8, 0.6504, 0.2969\}$
1375	$\{I_1, 0.1040, 0.0389\}, \{I_2, 0.8960, 0.0234\}$	2394	$\{I_7, 0.8917, 0.0298\}, \{I_8, 0.1083, 0.1260\}$
1325	$\{I_1, 0.3596, 0.0963\}, \{I_2, 0.6404, 0.0579\}$	2374.25	$\{I_7, 0.9928, 0.0022\}, \{I_8, 0.0072, 0.0093\}$
1376.4	$\{I_1, 0.0967, 0.0365\}, \{I_2, 0.9033, 0.0220\}$	2315.25	$\{I_6, 0.2945, 0.0986\}, \{I_7, 0.7055, 0.0642\}$
1205.9	$\{I_1, 0.9688, 0.0126\}, \{I_2, 0.0311, 0.0075\}$	2059.95	$\{I_5, 0.6004, 0.0700\}, \{I_6, 0.3996, 0.1139\}$
1132.25	$\{I_1, 0.6544, 0.0946\}$	2120.05	$\{I_5, 0.2930, 0.0604\}, \{I_6, 0.7070, 0.0983\}$

the market share price of SBI is discretized into eight equal-length intervals which are determined by the proposed CBP approach and these intervals are as follows.

$$u_1 = [1004.30,\ 1199.81],\ u_2 = [1199.81,\ 1395.31],$$
$$u_3 = [1395.31,\ 1590.82],\ u_4 = [1590.82,\ 1786.32]$$
$$u_5 = [1786.32,\ 1981.83],\ u_6 = [1981.83,\ 2177.33],$$
$$u_7 = [2177.33,\ 2372.84],\ u_8 = [2372.84,\ 2568.35]$$

The following eight fuzzy sets are defined in accordance of intervals.

$$F_i = \begin{cases} [1004.30 + (i-1)h,\ 1004.30 + i{*}h,\ 1004.30 + 2{*}i{*}h], \\ \ i = 1 \text{ to } 7,\ h = 195.50 \\ [1004.30 + (i-1)h,\ 1004.30 + i{*}h,\ 1004.30 + i{*}h], \\ \ i = 8, h = 195.50 \end{cases}$$

The membership grades to fuzzy sets are shown in Table 9. IFSs I_i ($i = 1, 2, \ldots, 8$) for the market share price of SBI time series data are also constructed using the construction algorithm of Singh et al. [40]. The intuitionistic fuzzified market share price of SBI is shown in Table 10. IFLR is established between two succeeding data points using Strong (α, β)-cut set approach where $\alpha = 0.60$ and $\beta = 0.074$ and shown in Table 11.

Table 11
IFLRs for market share price of SBI dataset.

SBI prices	IFLRs	SBI prices	IFLRs
1819.95	–	1355	$I_1 \to I_1, I_1 \to I_2$
1840	$I_4 \to I_4, I_4 \to I_5, I_5 \to I_5$	1891	$I_2 \to I_4, I_2 \to I_5$
1496.7	$I_4 \to I_2, I_4 \to I_3$	1935	$I_4 \to I_5, I_5 \to I_5$
1567.5	$I_2 \to I_3, I_3 \to I_3$	1840	$I_4 \to I_4, I_5 \to I_4, I_5 \to I_5$
1638.9	$I_2 \to I_3, I_3 \to I_3, I_3 \to I_4$	1886.9	$I_4 \to I_4, I_4 \to I_5$
1618	$I_3 \to I_3, I_3 \to I_4, I_4 \to I_3$	2235	$I_4 \to I_6, I_5 \to I_6$
1569.9	$I_3 \to I_2, I_3 \to I_3, I_4 \to I_2$	2500	$I_6 \to I_7, I_7 \to I_8$
1375	$I_2 \to I_2, I_3 \to I_1, I_3 \to I_2$	2394	$I_7 \to I_7, I_8 \to I_7, I_8 \to I_8$
1325	$I_1 \to I_2, I_2 \to I_1, I_2 \to I_2$	2374.25	$I_7 \to I_7, I_7 \to I_8, I_8 \to I_7$
1376.4	$I_1 \to I_2, I_2 \to I_1, I_2 \to I_2$	2315.25	$I_7 \to I_6, I_7 \to I_7, I_8 \to I_7$
1205.9	$I_1 \to I_1, I_2 \to I_1, I_2 \to I_2$	2059.95	$I_7 \to I_5, I_7 \to I_6$
1132.25	$I_1 \to I_1, I_2 \to I_1$	2120.05	$I_5 \to I_6$

The forecasted market share price of SBI using the proposed defuzzification technique. The forecasted market share price of SBI using the proposed forecasting method and various existing IFS-based forecasting approaches suggested by [21,22,25,27,29, 39] are presented in Table 12. Comparative analysis between the performance of the proposed forecasting method along with these existing IFS-based forecasting methods in terms of statistical error measures in forecasting the market share price of SBI is shown in Table 13.

Table 12
Forecasted market share price of SBI.

SBI Price	Joshi & Kumar [21]	Kumar & Gangwar [22]	Abhishek et al. [25]	Gautam & Singh [27]	Pant & Kumar [29]	Pant & Kumar [39]	Proposed Method
1819.95	–	–	–	–	-	–	–
1840	1777.8	1725.98	–	1725.33	1716	–	1854.58
1496.7	1865.7	1725.98	–	1725.33	1776	–	1493.07
1567.5	1531.5	1512.39	–	1562.94	1491	1461.95	1590.82
1638.9	1531.5	1512.39	–	1613.61	1491	1654.72	1659.36
1618	1777.8	1574.35	–	1546.05	1491	1686.13	1650.12
1569.9	1531.5	1574.35	–	1546.05	1491	1562.06	1461.28
1375	1531.5	1512.39	–	1360.26	1491	1485.04	1330.27
1325	1504.23	1305.52	–	1208.25	1542	1331.01	1322.61
1376.4	1504.23	1665.9	–	1700.99	1542	1358.94	1345.43
1205.9	1504.23	1305.52	–	1208.25	1542	1191.81	1262.16
1132.25	1258.23	1294.27	–	1258.92	1270	1206.9	1199.81
1355	1258.23	1294.27	–	1258.92	1270	1206.52	1309.73
1891	1504.23	1665.9	19000	1700.99	1542	1890.66	1884.08
1935	1865.71	2006.51	1999.99	2116.392	2041	1939.82	1981.83
1840	1883.93	2006.51	1999.99	1814.68	2041	1845.28	1852.4
1886.9	1865.71	1725.98	19000	1725.33	1776	1954.04	1884.08
2235	1865.71	2006.51	1999.99	2116.92	2041	2208.9	2177.33
2500	2142.04	2520	2483	2494.72	2200	2419.83	2466.7
2394	2245.65	2420	2416.3	2381.38	2422	2446.9	2417.89
2374.25	2191.75	2365.99	2350	2343.6	2422	2317.92	2433.09
2315.25	2191.75	2365.99	2200	2343.6	2422	2288.21	2300.5
2059.95	2142.04	2020	2200	1984.69	2200	2046.81	2079.58
2120.05	1883.93	2120	2150	2041.36	2041	2200.97	2177.33

Table 13
Comparison between performance of proposed and different methods in forecasting market share price of SBI.

Models	RMSE	MAPE	U	TS	R	R^2
Joshi and Kumar [21]	200.17	9.52	0.055	−4.21	0.882	0.7779
Kumar & Gangwar [22]	131.28	6.30	0.035	1.71	0.9446	0.8922
Abhishek et al. [25]	104.66	3.50	0.019	10.75	−0.5123	0.2624
Gautam et al. [27]	121.59	5.50	0.033	−3.05	0.9513	0.9049
Pant & Kumar [29]	177.45	9.35	0.048	1.33	0.8962	0.8031
Pant & Kumar [39]	62.30	2.83	0.016	−0.231	0.9886	0.9773
Proposed Method	**42.48**	**2.07**	**0.011**	**2.40**	**0.9944**	**0.9888**

6. Diebold Mariano test analysis

A Diebold Mariano test is used to test the significant difference between the performance of two different forecasting methods. A Diebold Mariano test with confidence levels of $\alpha = 0.001, 0.05$ is implemented on the absolute difference between actual and forecasted data in both experimental datasets of EUA and the market share price of SBI to test the significant difference between the proposed forecasting method with other existing IFTS forecasting models. In forecasting enrollments of the University of Alabama, results of the Diebold Mariano test (Table 14) indicate that the proposed forecasting method is significant over the method proposed by [21,24–27,29,37,39] while the difference is not significant with Kumar and Gangwar [22] in terms of p-value at 1% and 5% confidence level but less amount of error measure which confirms that proposed forecasting method is superior than that forecasting method.

Table 15 presents the results of the Diebold Mariano test in forecasting the market share price of SBI, the difference in the forecasting of the proposed method with [21,22,27,29] is significant. Although Diebold Mariano test indicates that the proposed forecasting method with those of Abhishek et al. [25] and Pant and Kumar [39] is not significant but less amount of error measure which confirms that the proposed forecasting method is superior in sensitive financial time series forecasting of the market share price of SBI.

Table 14
Diebold Mariano test analysis in forecasting enrollments of University of Alabama.

Compared Models	p-value	DM value
Joshi and Kumar [21]	0.0036^{***}	3.29
Kumar & Gangwar [22]	0.11	1.78
Wang et al. [24]	0.0037^{***}	3.27
Abhishek et al. [25]	0.016^{**}	2.61
Bisht & Kumar [37]	0.0095^{***}	2.89
Abhishek et al. [26]	0.039^{**}	2.19
Gautam & Singh [27]	0.036^{**}	2.23
Pant & Kumar [29]	0.01^{**}	2.82
Pant & Kumar [39]	0.011^{**}	2.18

**Denotes the significance at 5%, ***Denotes the significance at 1%.

Table 15
Diebold Mariano test analysis in forecasting market share price of SBI.

Compared Models	p-value	DM value
Joshi and Kumar [21]	0.0014^{***}	3.64
Kumar & Gangwar [22]	0.0043^{***}	3.17
Abhishek et al. [25]	0.166	1.49
Gautam & Singh [27]	0.02^{**}	2.5
Pant & Kumar [29]	0.0006^{***}	3.96
Pant & Kumar [39]	0.174	1.4

**Denotes the significance at 5%, ***Denotes the significance at 1%.

7. Conclusions

In the extant condition, it has been a challenging task for the decision makers to forecast the future value of an incidence due to the impedance of non-probabilistic uncertainty along with non-determinism in time series data. To consider the above issue in time series data, this research presented a novel IFTS forecasting method based on CBP and Strong (α, β)-cut approach. In this article, the appropriate IFLRs has been established by utilizing a Strong (α, β)-cut approach along with non-zero membership and non-membership values. A novel CBP approach is also proposed to determine the number of intervals and construction of intervals, which indicates its effective application to a diversity of time series data without the need of expert knowledge. Earlier in the IFTS forecasting method, only those IFSs are included for intuitionistic fuzzification having membership grade as the highest and non-membership grade was neglected without any reason. The novelty of the proposed IFTS forecasting method also addresses the issue of non-membership grade in intuitionistic fuzzification of time series data.

The proposed forecasting method experienced with a best statistical efficiency observation by using two well-known time series datasets with existing IFTS models. According to the findings of various measures (RMSE, MAPE, R, and R^2) for both considered time series, it can be extracted that the proposed model is dominant over its comparative models. The empirical analysis shows that the proposed model's RMSE and MAPE values are the lowest among other IFTS models, which demonstrates its superiority over its counterpart's models. The validity (correctness and unbiasedness) of the proposed method is confirmed by obtaining the desirable range of U (closer to 0) and TS value.

A significant difference between the proposed model and any of its comparative models is calculated by using the Diebold Mariano test and obtained results indicate that the proposed forecasting method is significant over its comparative models at 1% and 5% confidence levels. The proposed method can be used to forecast several fields of management and science. In future research, the proposed CBP approach can also be used to determine the number of intervals and partition of UOD in probabilistic, hesitant, and intuitionistic fuzzy set-based forecasting methods. Further, this work can also be extended to the high-order IFTS forecasting method.

CRediT authorship contribution statement

Manish Pant: Conceptualization, Methodology, Investigation, Writing – original draft. **Kamlesh Bisht:** Conceptualization, Methodology, Investigation, Writing – original draft. **Seema Negi:** Conceptualization, Methodology, Visualization, Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

References

[1] L.A. Zadeh, Fuzzy Set. Inf. Control 8 (1965) 338–353, http://dx.doi.org/10.1109/TFUZZ.2012.2188897.

[2] Q. Song, B.S. Chissom, Forecasting enrollments with fuzzy time series — Part I, Fuzzy Sets Syst. 54 (1993) 1–9, http://dx.doi.org/10.1016/0165-0114(93)90355-L.

[3] Q. Song, B.S. Chissom, Fuzzy time series and its models, Fuzzy Sets Syst. 54 (1993) 269–277.

[4] Q. Song, B.S. Chissom, Forecasting enrollments with fuzzy time series — part II, Fuzzy Sets Syst. 62 (1994) 1–8, http://dx.doi.org/10.1016/0165-0114(94)90067-1.

[5] S.M. Chen, Forecasting enrollments based on fuzzy time series, Fuzzy Sets Syst. 81 (1996) 311–319, http://dx.doi.org/10.1016/0165-0114(95)00220-0.

[6] K. Huarng, Effective lengths of intervals to improve forecasting in fuzzy time series, Fuzzy Sets Syst. 123 (2001) http://dx.doi.org/10.1016/S0165-0114(00)00057-9.

[7] H.J. Teoh, C.H. Cheng, H.H. Chu, J.S. Chen, Fuzzy time series mode based on probabilistic approach and rough set rule induction for empirical research in stock markets, Data Knowl. Eng. 67 (2008) 103–117, http://dx.doi.org/10.1016/j.datak.2008.06.002.

[8] C.H. Cheng, G.W. Cheng, J.W. Wang, Multi-attribute fuzzy time series method based on fuzzy clustering, Expert Syst. Appl. 34 (2008) 1235–1242, http://dx.doi.org/10.1016/J.ESWA.2006.12.013.

[9] K. Huarng, T.H.K. Yu, Ratio-based lengths of intervals to improve fuzzy time series forecasting, IEEE Tran. Syst. Man Cybern. B 36 (2006) 328–340, http://dx.doi.org/10.1109/TSMCB.2005.857093.

[10] S.M. Chen, N.Y. Chung, Forecasting enrollments using high-order fuzzy time series and genetic algorithms, Int. J. Intell. Syst. 21 (2006) 485–501, http://dx.doi.org/10.1002/INT.20145.

[11] I.H. Kuo, S.J. Horng, T.W. Kao, T.L. Lin, C.L. Lee, Y. Pan, An improved method for forecasting enrollments based on fuzzy time series and particle swarm optimization, Expert Syst. Appl. 36 (2009) 6108–6117, http://dx.doi.org/10.1016/j.eswa.2008.07.043.

[12] E. Bas, C. Grosan, E. Egrioglu, U. Yolcu, High order fuzzy time series forecasting method based on pi-sigma neural network, Eng. Appl. Artif. Intell. 72 (2018) 350–356.

[13] S. Zeng, S.M. Chen, M.O. Teng, Fuzzy forecasting based on linear combinations of independent variables, subtractive clustering algorithm and artificial bee colony algorithm, Inf. Sci. 484 (2019) 350–366.

[14] P. Singh, A novel hybrid time series forecasting model based on neutrosophic-PSO approach, Int. J. Machine Learn. Cybern. 11 (2020) 1643–1658, http://dx.doi.org/10.1007/S13042-020-01064-Z.

[15] M. Pant, S. Kumar, Hesitant fuzzy sets, particle swarm optimization and support vector machine-based hybrid method for fuzzy time series forecasting, Granul. Comput. (2021) 1–19, http://dx.doi.org/10.1007/s41066-021-00265-3.

[16] N. Kumar, S. Susan, Particle swarm optimization of partitions and fuzzy order for fuzzy time series forecasting of COVID-19, Appl Soft Comput (2021) 107611.

[17] J. Sullivan, W.H. Woodall, A comparison of fuzzy forecasting and Markov modeling, Fuzzy Sets Syst. 64 (1994) 279–293.

[18] K. Huarng, T.H.K, Yu the application of neural networks to forecast fuzzy timeseries, Physica A 363 (2006) 481–491.

[19] G. Goyal, D.C.S. Bisht, Strong α-cut and associated membership-based modeling for fuzzy time series forecasting, Int. J. Model Simul. Sci. Comput. 12 (2021) 2050067, http://dx.doi.org/10.1142/S1793962320500671.

[20] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst. 20 (1986) 87–96.

[21] B.P. Joshi, S. Kumar, Intuitionistic fuzzy sets based method for fuzzy time series forecasting, Cybern. Syst. 43 (2012) 34–47, http://dx.doi.org/10.1080/01969722.2012.637014.

[22] S. Kumar, S.S. Gangwar, Intuitionistic fuzzy time series: An approach for handling nondeterminism in time series forecasting, IEEE Trans. Fuzzy Syst. 24 (2016) 1270–1281, http://dx.doi.org/10.1109/TFUZZ.2015.2507582.

[23] S.S. Gangwar, S, Kumar probabilistic and intuitionistic fuzzy sets-based method for fuzzy time series forecasting, Cybern. Syst. 45 (2014) 349–361, http://dx.doi.org/10.1080/01969722.2014.904135.

[24] Y. Wang, Y. Lei, X. Fan, Y. Wang, Intuitionistic fuzzy time series forecasting model based on intuitionistic fuzzy reasoning, Math. Prob. Eng. (2016) http://dx.doi.org/10.1155/2016/5035160.

[25] Abhishekh, S.S. Gautam, S.R. Singh, A score function-based method of forecasting using intuitionistic fuzzy time series, New Math. Nat. Comput. 14 (2018) 91–111, http://dx.doi.org/10.1142/S1793005718500072.

[26] Abhishekh, S.S. Gautam, S.R. Singh, A new method of time series forecasting using intuitionistic fuzzy set based on average-length, J. Ind. Prod. Eng. 37 (2020) 175–185.

[27] Gautam, S.R. Singh, A modified weighted method of time series forecasting in intuitionistic fuzzy environment, OPSEAECH (2020) http://dx.doi.org/10.1007/s12597-020-00455-8.

[28] M. Pant, A.K. Shukla, S. Kumar, A novel method to optimize interval length for intuitionistic fuzzy time series, in: A. Tiwari, K. Ahuja, A. Yadav, J.C. Bansal, K. Deep, A.K. Nagar (Eds.), Soft Computing for Problem Solving. Advances in Intelligent Systemsand Computing, vol. 1393, 2021, pp. 55–62, http://dx.doi.org/10.1007/978-981-16-2712-5_5.

[29] M. Pant, S. Kumar, Particle swarm optimization and intuitionistic fuzzy set-based novel method for fuzzy time series forecasting, Granul. Comput. 7 (2021) 285–303, http://dx.doi.org/10.1007/s41066-021-00300-3.

[30] E. Egrioglu, U. Yolcu, E. Bas, Intuitionistic high-order fuzzy time series forecasting method based on pi-sigma artificial neural networks trained by artificial bee colony, Granul. Comput. 4 (2019) 639–654, http://dx.doi.org/10.1007/s41066-018-00143-5.

[31] C. Kocak, E. Egrioglu, E. Bas, A new deep intuitionistic fuzzy time series forecasting method based on long short-term memory, J. Supercomput. 77 (2020) 6178–6196, http://dx.doi.org/10.1007/S11227-020-03503-8.

[32] C. Kocak, E. Egrioglu, E. Bas, A new explainable robust high order intuitionistic fuzzy time series method, Soft. Comput. (2021) 1–14, http://dx.doi.org/10.1007/s00500-021-06079-4.

[33] R.M. Pattanayak, H.S. Behera, S. Panigrahi, A novel probabilistic intuitionistic fuzzy set based model for high order fuzzy time series forecasting, Eng. Appl. Arti. Intell. 99 (2021) 104136, http://dx.doi.org/10.1016/j.engappai.2020.104136.

[34] K.K. Gupta, S. Kumar, Probabilistic intuitionistic fuzzy set based intuitionistic fuzzy time series forecasting method, Int. Conf. Math. Model Sci. Comput. (2018) 315–324.

[35] M. Pant, A.K. Shukla, S. Kumar, Novel intuitionistic fuzzy time series modeling to forecast the death cases of COVID-19 in India, in: Y.D. Zhang, T. Senjyu, C. So-In, A. Joshi (Eds.), Smart Trends in Computing and Communications, in: Lecture Notes in Networks and Systems, vol. 286, 2022, pp. 525–531, http://dx.doi.org/10.1007/978-981-16-4016-2_49.

[36] Nik Muhammad Farhan Hakim Nik Badrul Alam Alam, N. Ramli, A.H. Mohd, Intuitionistic fuzzy set-based time series forecasting model via delegeration of hesitancy degree to the major grade de-i-fuzzification and arithmetic rules based on centroid defuzzification, in: J Phys: Conference Series, 2020, http://dx.doi.org/10.1088/1745-6596/1988/1/012014.

[37] K. Bisht, S. Kumar, Intuitionistic fuzzy set-based computational method for financial time series forecasting, Fuzzy Inf. Eng. 10 (2018) 307–323, http://dx.doi.org/10.1080/16168658.2019.1631557.

[38] S. Kumar, A modified weighted fuzzy time series model for forecasting based on two- factors logical relationship, Int. J. Fuzzy Syst. 21 (2019) 1403–1417, http://dx.doi.org/10.1007/s40815-019-00652-8.

[39] S. Pant, S. Kumar, IFS and SODA based computational method for fuzzy time series forecasting, Expert Syst. Appl. (2022) 118213, http://dx.doi.org/10.1016/j.eswa.2022.118213.

[40] A. Singh, D.K. Joshi, S. Kumar, A novel construction method of intuitionistic fuzzy set from fuzzy set and its application in multi-criteria decision-making problem, in: Advanced Computing and Communication Technologies, Springer, Singapore, 2019, pp. 67–75.